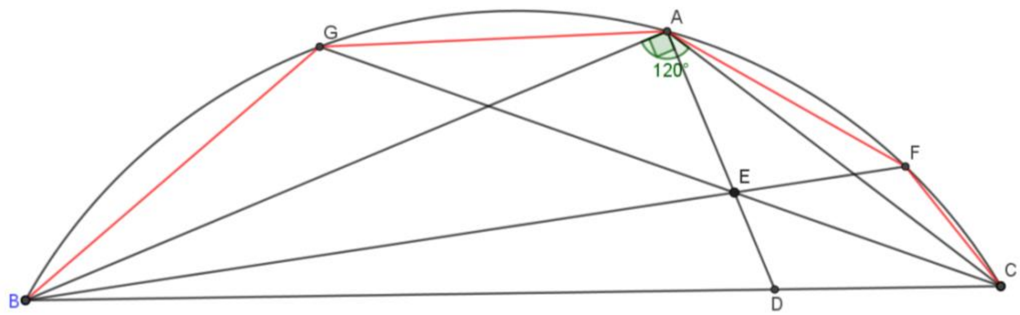
# Cash Award Question of Jan-2026




**In the picture, ΔABC is inscribed in its circumcircular arc with ∠BAC = 120°. D is a point on BC such that ∠BAD = 90°. E is a random point on AD. BE & CE are joined and produced to meet the circumcircular arc at F & G respectively.**

**Prove:** $\frac{AF}{FC} = 2\left(\frac{AG}{GB}\right)$

**Question framed by**
**DR. M. RAJA CLIMAX, IRS**
**Asst. Commissioner of Customs & GST (Rtd),**
**Madurai, Tamil Nadu, India.**

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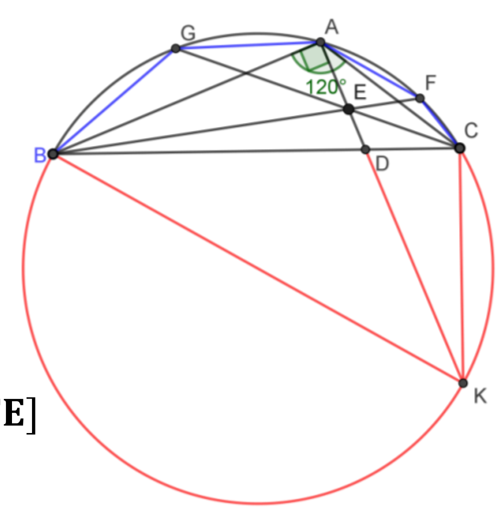
# Author's Solution Jan-2026

**Construction:**

Draw the circumcircle of $\Delta\, ABC$, Extend the line AD to meet the circumcircle at K.
Join BK & CK.




**Proof:**

**ΔBEG ~ ΔCEF**

**[(AA Similarity),** $\angle\mathbf{BEG} = \angle\mathbf{CEF}\ \&\ \angle\mathbf{BGE} = \angle\mathbf{CFE}$**]**

$\Rightarrow \frac{BG}{CF} = \frac{BE}{CE}$ ----------------- (1)

**Similarly ΔFAE ~ ΔKBE**

$\Rightarrow \frac{FA}{KB} = \frac{AE}{BE}$ ----------------- (2)

**ΔKCE ~ ΔGAE**

$\Rightarrow \frac{KC}{GA} = \frac{CE}{AE}$ ----------------- (3)

**Multiply (1),(2)&(3)→** $\frac{BG}{CF} \times \frac{FA}{KB} \times \frac{KC}{GA} = \frac{BE}{CE} \times \frac{AE}{BE} \times \frac{CE}{AE} = 1$

$\Rightarrow \frac{AF}{FC} \times \frac{CK}{BK} \times \frac{BG}{GA} = 1$ -----------------(I)

As $\angle BAC = 120° \,\&\, \angle BAK = 90°$

$\Rightarrow \angle KAC = 30° = \angle KBC$ (Angle subtended by CK)

As $\angle BAK = 90°$, BK is diameter $\Rightarrow \angle BCK = 90° \,\&\, \angle CBK = 30°$.

$\therefore \Delta\, BCK$ is special triangle (30°, 60°, 90°) BK = 2R , CK = R ,

then equation (I) becomes $\frac{AF}{FC} \times \frac{R}{2R} \times \frac{BG}{GA} = 1$

Hence $\frac{AF}{FC} = 2\, \frac{AG}{GB}$ ----------- Proved.

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Solution given by
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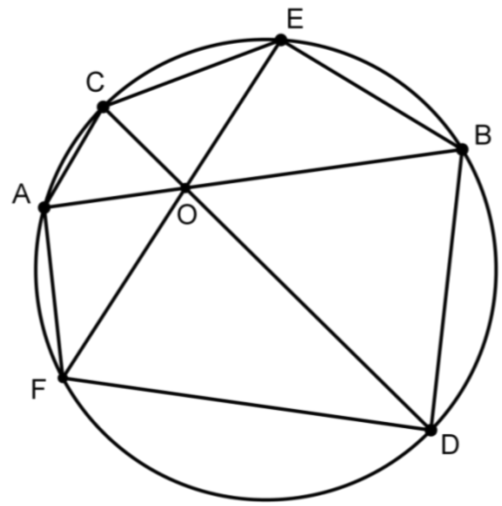
**Also, see the following Theorem for reference…**

## The Concurrent Chords Theorem:

AB, CD & EF are any three chords of a circle concurrent at O. These chords form the chords AF, FD, DB, BE, EC & CA along the circumference of the circle. The Concurrent Chords Theorem says, $\frac{AC}{CE} \times \frac{EB}{BD} \times \frac{DF}{FA} = 1$ ie ACxDFxBE = AFxBDxCE.

**Proof:**




ΔAOC ~ ΔDOB

$\Rightarrow \frac{AC}{BD} = \frac{OA}{OD}$ ------------------ (1)

ΔBEO ~ ΔFAO

$\Rightarrow \frac{BE}{AF} = \frac{OE}{OA}$ ------------------ (2)

ΔFDO ~ ΔCEO

$\Rightarrow \frac{FD}{CE} = \frac{OD}{OE}$ ------------------ (3)

(1),(2)&(3)→

$\frac{AC}{BD} \times \frac{BE}{AF} \times \frac{FD}{CE} = \frac{\cancel{OA}}{\cancel{OD}} \times \frac{\cancel{OE}}{\cancel{OA}} \times \frac{\cancel{OD}}{\cancel{OE}} = 1$

$\Rightarrow$ ACxBExFD = AFxBDxCE

Or $\frac{AC}{CE} \times \frac{EB}{BD} \times \frac{DF}{FA} = 1$ ------------- Proved

**This Concurrent Chords Theorem was discovered by :**

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